QUIZ 1 - CALCULUS 2 (2020/11/26)

1. (10 pts) Find f.

   (a) (5 pts) $f'(x) = e^x - \sin x, \quad f(\pi) = 0$.

   (b) (5 pts) $\int_1^x f(t)e^{-t}\, dt = 4\tan^{-1} x - \pi$. (Hint: FTC1)

   Solution:

   (a)

$$f(x) = e^x + \cos x + C$$
$$f(\pi) = e^\pi - 1 + C = 0$$
$$C = 1 - e^\pi$$
$$f(x) = e^x + \cos x + 1 - e^\pi$$

   (b)

$$\frac{d}{dx}\int_1^x f(t)e^{-t}\, dt = f(x)e^{-x}$$
$$\frac{d}{dx}(4\tan^{-1} x - \pi) = \frac{4}{1+x^2}$$
$$f(x) = \frac{4e^x}{1+x^2}$$

2. (10 pts) Evaluate the integral by

   Step 1: finding an antiderivative of the integrand,

   Step 2: using FTC2.

   (a) (5 pts) $\int_1^4 \left(\frac{1}{x} + \frac{1}{\sqrt{x}} + \frac{1}{x\sqrt{x}}\right)\, dx$.

   (b) (5 pts) $\int_0^{\pi/4} \left(3\sin x + 4\cos x - 5\sec^2 x\right)\, dx$.

   Solution:

   (a)

$$\int_1^4 \left(\frac{1}{x} + \frac{1}{\sqrt{x}} + \frac{1}{x\sqrt{x}}\right)\, dx$$
$$= \int_1^4 \left(x^{-1} + x^{-1/2} + x^{-3/2}\right)\, dx$$
$$= \left[\ln|x| + 2x^{1/2} - 2x^{-1/2}\right]_1^4$$
$$= \ln 4 + 4 - 1 - \ln 1 - 2 + 2 = 3 + 2\ln 2$$

(b)

$$\int_0^{\pi/4} \left(3\sin x + 4\cos x - 5\sec^2 x\right)\, dx$$
$$= \left[-3\cos x + 4\sin x - 5\tan x\right]_0^{\pi/4}$$
$$= \frac{-3}{\sqrt{2}} + \frac{4}{\sqrt{2}} - 5 + 3 - 0 + 0 = \frac{1}{\sqrt{2}} - 2$$

Grading scheme: For all four problems, (-2 pts) for each mistake until there are no points.